Statistics

Winter 2022

Lecture 9

Live QZ 2

L1	L2
1	.05
2	.15
3	.2
4	.25
5	.25
6	.1

Use L1 & L2 to find

$\bar{x} = 3.8$

S = Blank

$n = 1$

$\boxed{\text{STAT}}\ \boxed{\rightarrow}$ CALC

$\boxed{\text{1-Var Stats}}$

List: L1

FreqList: L2

$\boxed{\text{Calculate}}$

1-Var Stats

L1, L2

$\boxed{\text{enter}}$

Ch. 1:

Data
- 1) Qualitative
  (Non-Numerical)
- 2) Quantitative
  (Numerical)
  - 1) Discrete
    (Countable)
    # flights LA to SF
  - 2) Continuous
    (Measureable)
    flight time LA to SF

Let x be a discrete random Variable with Prob. dist of $P(x)$

It is a method to give us all prob. for all Possible outcomes

- In the form of a table (Can be created by our Knowledge of Prob.)
- In the form of a graph
- By the way of a formula.

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Some Prob. Rules

1) $0 \le P(x) \le 1$

2) $\sum P(x) = 1$

3) $P(x) = 1 \iff$ Sure event

4) $P(x) = 0 \iff$ Impossible event

5) $0 < P(x) \le .05 \iff$ Rare event

Mean μ "mu" — $\mu = \sum x\,p(x)$

Variance σ^2 "sigma"^{2} — $\sigma^2 = \sum x^2 p(x) - \mu^2$

Standard Deviation σ "sigma" — $\sigma = \sqrt{\sigma^2}$

Consider the chart below

x	$P(x)$	$x\,P(x)$	$x^2P(x)$
1	.1	.1	.1
2	.2	.4	.8
3	.5	1.5	4.5
4	.2	.8	3.2

① Verify $\sum P(x) = 1$

$.1 + .2 + .5 + .2 = 1$ ✓

$\mu = \sum x\,p(x)$

$= .1 + .4 + 1.5 + .8$

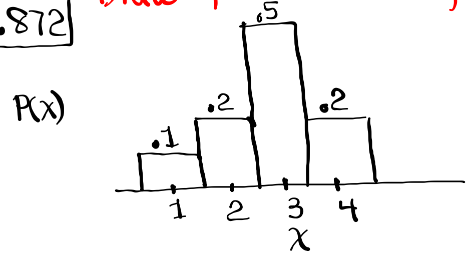
$= \boxed{2.8}$

$\sigma^2 = \sum x^2 p(x) - \mu^2$

$= .1 + .8 + 4.5 + 3.2 - 2.8^2 = \boxed{.76}$

$\sigma = \sqrt{\sigma^2} = \sqrt{.76} = \boxed{.872}$

Draw prob. dist. Histogram



Using TI

$x \to L1$

$P(x) \to L2$

L1	L2
1	.1
2	.2
3	.5
4	.2

$\boxed{\text{STAT}}$ $\boxed{\to}$ CALC

$\boxed{\text{1-Var Stats}}$

List: L1

FreqList: L2

$\boxed{\text{Calculate}}$

L1, L2 $\boxed{\text{Enter}}$

$\mu = \bar{x} = 2.8$

$\sigma = \sigma_x = .872$

$n = 1$

How to find σ^2:

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{4{:}\ \sigma_x}$ $\boxed{x^2}$

$\boxed{\text{Enter}}$ $\boxed{\sigma^2 = .76}$ exact

Convert to fraction

$\boxed{\text{MATH}}$ $\boxed{1{:}\blacktriangleright\text{Frac}}$ $\boxed{\text{Enter}}$ $\boxed{\sigma^2 = \frac{19}{25}}$

Consider the chart below for x with prob. dist $P(x)$:

x	$P(x)$
1	.08
2	.15
3	.27
4	.35
5	.15

1) Find $P(x=5)$

$P(x=5) = 1 - [.08 + .15 + .27 + .35]$

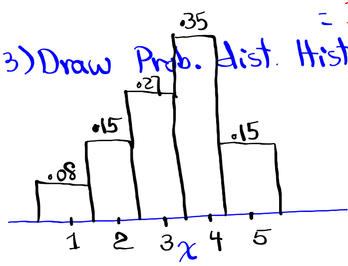
Total Prob.

$= 1 - .85 = \boxed{.15}$

2) Find $P(x \geq 2)$

$= 1 - .08 = \boxed{.92}$

3) Draw Prob. dist. Histogram



$x \to L1$, $P(x) \to L2$

1-Var Stats with L1, L2

4) Find

$\mu = \bar{x} = 3.34$

$\sigma = \sigma_x = 1.142$

σ^2(exact) $\frac{3261}{2500}$ ✓

$\boxed{\text{VARS}}$ $\boxed{5{:}}$ $\boxed{4{:}\ \sigma_x}$

$\boxed{x^2}$ $\boxed{\text{Math}}$ $\boxed{1{:}\blacktriangleright\text{Frac}}$

$\boxed{\text{Enter}}$

A piggy bank has 8 nickels and 2 Dimes.
Shake it to get 2 Coins.

DD → 20¢

$P(20¢) = \frac{2}{10} \cdot \frac{1}{9} = \frac{2}{90} = \frac{1}{45}$

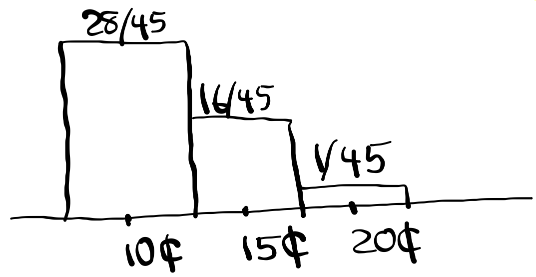
DN
ND → 15¢

$P(15¢) = 2\left(\frac{2}{10} \cdot \frac{8}{9}\right) = \frac{32}{90} = \frac{16}{45}$

NN → 10¢

$P(10¢) = \frac{8}{10} \cdot \frac{7}{9} = \frac{56}{90} = \frac{28}{45}$

Total(¢)	P(total(¢))
20¢	1/45
15¢	16/45
10¢	28/45



Find

$\mu = \bar{x} = 12$

$\sigma = \sigma_x = 2.667$

$\sigma^2(\text{exact}) = \boxed{\frac{64}{9}}$

Total → L1

P(Total) → L2

1-Var stats with L1 & L2

VARS | 5: | 4: | $\bar{x}^2$

MATH | 1: ▸Frac | Enter

Application:

20 students

Each bought one Ticket

for $15

One Ticket is drawn,

Winner gets a Calc worth $100.

Expected Value per ticket

for the house

Net Profit	P(Net profit)
15 − 100	$\frac{1}{20}$
15 − 0	$\frac{19}{20}$

for the house

Net profit → L1

P(Net profit) → L2

1-Var Stats with L1 & L2

Expected Value = $\mu = \bar{x}$ → $10

I make $10 per ticket.

You are having a surgery.

Prob. that you don't make is is .5%

You buy a policy for $500

Insurance company pays out $100,000

if you don't make it.

Expected Value per policy sold for insurance Co.

Net Profit	P(Net Profit)
500 − 100000	.5% = .005
500 − 0	99.5% = .995

E.V. = $\mu = \bar{x}$

Net profit → L1

P(Net Profit) → L2

1-Var Stats

L1 & L2

E.V. = $\mu = \bar{x}$ = 0 ← Break-even

Change $500 → $750

and recalculate.

E.V. $250

Net Profit / Policy Sold.

Binomial Prob. Dist:

1) n independent events (trials)

2) Each event has only two outcomes.

$P(\text{Success}) = p$ $\quad$ $P(\text{Failure}) = q$

$p + q = 1, \quad q = 1 - p$

3) p & q remain unchanged for all n trials.

4) $x \to$ # of successes

$n - x \to$ # of failures

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

Binomial Prob. dist. Formula

ex: Consider a binomial Prob. dist with $n = 10$, and $p = .6$. Let x be # of successes.

Find P(exactly 7 successes)

$$= P(x=7) = {}_{10}C_7 \cdot (.6)^7 \cdot (.4)^3 \approx \boxed{.215}$$

n $\quad$ x $\quad$ p $\quad$ $q = 1 - p$

Find P(exactly 5 successes)

$$= P(x=5) = {}_{10}C_5 \cdot (.6)^5 \cdot (.4)^5 = \boxed{.201}$$

n $\quad$ x $\quad$ p $\quad$ q

You are making random guesses on a true/false test with 30 questions.

$_nC_x \cdot p^x \cdot q^{n-x}$

$n=30$, $p=.5$, $q=.5$

P(guess exactly 12 correct answers)

$P(x=12) = {}_{30}C_{12} \cdot (.5)^{12} \cdot (.5)^{18} = \boxed{.081}$ ✓

Using TI:

$\boxed{2nd}$ $\boxed{VARS}$ ↓↓ $\boxed{binompdf(}$

with Menu

Trials: 30

P: .5

x-value 12

$\boxed{Paste}$ $\boxed{Enter}$

30, .5, 12 $\boxed{Enter}$

$\boxed{)}$ $\boxed{)}$

You are randomly guessing on a multiple-choice exam with 40 questions.

Each question has 4 choices, but only one correct choice.

$n=40$, $p=\frac{1}{4}=.25$ $q=\frac{3}{4}=.75$

Find P(guess $\underbrace{\text{exactly 15}}_{x=15}$ correct answers)

$P(x=15)=$ binompdf(40, .25, 15) $= \boxed{.028}$

Find P(guess $\underbrace{\text{exactly 20}}_{x=20}$ Correct answers)

$P(x=20)=$ binompdf(40, .25, 20) $= \boxed{3.976 \times 10^{-4}}$

$P(x=a) =$ binompdf(n, p, a)

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A fair coin is tossed 100 times.

Success is to land tails

$n = 100$

$p = .5$

$q = .5$

P(land at most 60 tails)

$x \le 60$

$P(x \le 60) = P(x=60) + P(x=59) + P(x=58) + \cdots + P(x=0)$

$= \text{binomcdf}(\underset{n}{100}, \underset{p}{.5}, \underset{x}{60})$

$= \boxed{.982}$

P(land fewer than 50 tails)

$x < 50$

$P(x < 50) = P(x \le 49) = \text{binomcdf}(100, .5, 49)$

$= \boxed{.460}$ ✓

$P(x = a) = \text{binompdf}(n, p, a)$

$P(x \le a) = \text{binomcdf}(n, p, a)$

Prob. of passing a math class per student is .7.

Let's randomly select 50 students,

Find P(exactly 35 pass)

$P(x = 35) = \text{binompdf}(50, .7, 35) = \boxed{.122}$

Find P(fewer than 35 pass)

$P(x < 35) = P(x \le 34) = \text{binomcdf}(50, .7, 34) = \boxed{.431}$

Find P(at least 30 pass)

$P(x \ge 30) = 1 - P(x \le 29) = 1 - \text{binomcdf}(50, .7, 29)$

we don't want | we want

29 30

$= \boxed{.952}$

$P(x=a) = \text{binompdf}(n, P, a)$

$P(x \leq a) = \text{binomcdf}(n, P, a)$

$P(x \geq a) = 1 - \text{binomcdf}(n, P, a-1)$

Consider a binomial prob. dist with $n = 400$ and $P = .8$, x is # of Successes.

$P(x = 325) = \text{binompdf}(400, .8, 325) = \boxed{.042}$

$P(x \leq 325) = \text{binomcdf}(400, .8, 325) = \boxed{.752}$

$P(x \geq 325) = 1 - \text{binomcdf}(400, .8, 324) = \boxed{.290}$

$P(x=a) = \text{binompdf}(n, P, a)$

$P(x \leq a) = \text{binomcdf}(n, P, a)$

$P(x \geq a) = 1 - \text{binomcdf}(n, P, a-1)$

$P(a \leq x \leq b) = \text{binomcdf}(n, P, b) - \text{binomcdf}(n, P, a-1)$

Reduce by 1.

CDC says 60% of all Americans are vaccinated.
Let's randomly select 150 Americans.
Find the prob. that $\boxed{\text{between 85 and 100}}$ of them are vaccinated.

inclusive

$n = 150$

$P = .6$

$P(85 \le x \le 100)$

$= \text{binomcdf}(150, .6, 100) - \text{binomcdf}(150, .6, 84)$

Reduce by 1

$= \boxed{.782}$

Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard Deviation $\sigma = \sqrt{\sigma^2}$

Binomial Prob. Dist.

100 Newborn babies are randomly Selected. Assume girls are success.

$n=100$ $\quad p=.5$ $\quad q=.5$

$\mu=np=100(.5)=50$

$\sigma^2=npq=100(.5)(.5)=25$

$\sigma=\sqrt{\sigma^2}=\sqrt{25}=5$

68% Range $\Rightarrow \mu\pm\sigma=50\pm5\Rightarrow\boxed{45\text{ to }55}$

Usual Range $\Rightarrow \mu\pm2\sigma=50\pm2(5)\Rightarrow\boxed{40\text{ to }60}$

"95% Range"

Live QZ 3

x (L1)	$P(x)$ (L2)
1	.05
2	.1
3	.15
4	.2
5	.25
6	.25

Find

$\mu=4.25$

$\sigma=1.479$

$\sigma^2(\text{exact})=\frac{35}{16}=2.1875$